2.1 Notes and Examples   Name: Block: Seat:

The Derivative and the Tangent Line Problem

Consider two points on a curve, $(a, f(a)$ and $(b, f(b))$. If you connect them you form a ______________ line. The slope of of that secant line is the average rate of f between a and b. As those two locations a and b get closer and closer to each other, the line connecting them starts to look more and more like a

______________ line, where is touches f at one place. The slope of **that** line now looks like the rate at a

single moment, or an ______________rate.

Definition of a Tangent Line with Slope m

$$m = \lim_{\Delta \to 0} \frac{\Delta y}{\Delta x} =$$

which we also often see as

$$m = \lim_{h \to 0} =$$

If this limit exists, then the line passing through $(c, f(c))$ with slope m is the **tangent line** to the graph of f at the point $(c, f(c))$.

1. Find the slope of the graph of $f(x) = 2x - 3$ at the point $(2, 1)$

Definition of the Derivative of a Function
The **derivative** of f at $x = a$ is given by

$$f'(a) = \left.\frac{dy}{dx}\right|_{x=a} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Other common notations for derivatives include $y' = \dfrac{d}{dx}[f(x)] = D_x[y]$

Theorem: If it is Differentiable it is _________________________

Proof: Using the definition of Continuity at a point:

1. Is $f(a)$ defined?

2. Does the limit exist?

3. Does $f(a) = \lim_{x \to a} f(x)$?

Is the converse true? If f is continuous, must it be differentiable?

Counterexample:

2. Let f be a function which satisfies $f(2+h) - f(2) = 5h - 3h^2 + 9h^3$ for all real numbers h. Find $f'(2)$.

3. Use the limit definition to find the derivative of $f(x)$:
   (a) If $f(x) = x^2 + 2x$

   (b) If $f(x) = \sqrt{x}$

   (c) If $f(x) = x^3$

4. Let $y = \frac{2}{t}$. Find $\frac{dy}{dt}$.

5. Given $\lim_{x \to 5} \frac{f(x) - f(5)}{x - 5} = 3$, for each of the following select MUST be true, MIGHT be true, or NEVER be true:
   (a) $f'(5) = 3$
   (b) $f'(5) = 0$
   (c) $f(5) = 3$
   (d) f is continuous at $x = 0$
   (e) f is continuous at $x = 5$
   (f) $\lim_{x \to 5} f(x) = f(5)$

6. Find $\lim_{x \to 3} \frac{x^2 + 2x - 15}{x - 3}$ *Hint:* does this remind you of 3(a)?

7. Use the limit definition of derivative $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$, and the theorems $\lim_{h\to 0} \frac{\sin h}{h} = 1$, $\lim_{h\to 0} \frac{1-\cos h}{h} = 0$ to prove the following:

   (a) $\frac{d}{dx}[\sin x] = \cos x$

   (b) $\frac{d}{dx}[\cos x] = -\sin x$

AP Style

8. Water is flowing into a tank over a 24 hour period. The amount of water in the tank is modeled by a differentiable function W for $0 \leq t \leq 24$, where t is measured in hours and $W(t)$ is measured in gallons. Values of $W(t)$ at selected values of time t are shown in the table below.

t (hours)	0	4	8	12	16	20	24
$W(t)$ in (gallons)	150	184	221	257	294	327	357

(a) Use the data in the table to find $W(8)$. Using appropriate units, explain the meaning of your answer.

(b) Use the data in the table to find $W^{-1}(257)$. Using appropriate units, explain the meaning of your answer.

(c) Use the data in the table to find an approximation for $W'(15)$. Show the computations that lead to your answer. Using appropriate units, explain the meaning of your answer.

(d) Use the data in the table to find the average rate of change of $W(t)$ over the time period $4 \leq t \leq 20$. Show the computations that lead to your answer.

(e) For $0 < t < 24$, must there be a time t when the tank contains 265 gallons of water? Justify your answer.

(f) A model for the amount of water in the tank is given by $A(t) = \dfrac{1}{225}(-t^3 + 30t^2 + 1800t + 33750)$ where $A(t)$ is measured in gallons and t is measured in hours. Use your calculator to find $A'(15)$.

Hint: Use $\boxed{\text{MATH}}$ $\boxed{\text{8. Deriv}}$.
If you have an TI-83 the order of the parameters are nDeriv(Y_1, X, 15)

(g) A model for the amount of water in the tank is given by $A(t) = \dfrac{1}{225}(-t^3 + 30t^2 + 1800t + 33750)$ where $A(t)$ is measured in gallons and t is measured in hours. Use your calculator to find the average rate of change over the time period $4 \leq t \leq 20$ hours.

Hint: Recall the average rate of change between $x = a$ and $x = b$ is the slope of the secant line:

$$\frac{rise}{run} = \frac{f(b) - f(a)}{b - a}$$